Name Fatima Alzarooni, ID 94188

20/20.

Quiz I

Ayman Badawi

QUESTION 1. (8 points) i) Find the gcd(121, 220), show the work

$220 = 1 \cdot 121 + 99$

$121 = 1 \cdot 99 + 22$

$99 = 4 \cdot 22 + 11$

ans ← 11) $22 = 2 \cdot 11 + 0$ → stop

gcd (121 , 220) = 11

ii) Find the LCM[121, 220], show the work

$= \frac{(121)(220)}{gcd(121, 220)} = \frac{(121)(220)}{11} = 2420$

QUESTION 2. (6 points) i) Solve $\overset{a}{8}x = \overset{b}{12}$ over the planet Z_{20} n

gcd (a, n) = gcd (8 , 20) = 4

Is 4 a factor of 12 ? yes ⟹ 4 solutions

+ $d = \frac{n}{gcd(a, n)} = \frac{20}{4} = 5$

solution set = { 4 , 9 , 14 , 19 }

ii) Let $D = \{1 \leq a < 500\}$. Find $|D|$. gcd (a , 500) = 1

n = 500 = 250 × 2 = 25 × 10 × 2 = 5 × 5 × 5 × 2 × 2 = $5^3 \times 2^2$

$|D| = \phi(500) = 5^2(5-1) \times 2^1(2-1) = 200$

QUESTION 3. i) Find -47 *mod* 19

= 19 − (47 mod 19) = 19 − 9 = 10 ← ans

ii) Find 203 *mod* 23

$203 = 8 \cdot 23 + 19$, $23 \cdot 8 = 184$, $203 - 184 = 19$ ans

203 mod 23 = 19

$125 = 5^3$

$5 \quad 5 \quad 5$

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$\frac{20}{20}$

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$\phi(5^3) = 5^2(5-1) = 100$

Quiz II

Ayman Badawi

$\gcd(2,125) = 1 \Rightarrow 2^{\phi(n)} \equiv 1 \mod 125$

~~$2006 = 16(125)+6$~~

QUESTION 1. (4 points) Find $2^{2006} (mod\ 125)$, show the work

$2^{2006} \pmod{125} = \cancel{2^{16(125)+6}} \ \cancel{(\text{mod } 125)}$

$\equiv 2^{20(100)+6} \pmod{125} \equiv 1 \cdot 2^6 \pmod{125}$

$\equiv 64 \pmod{125} = 64.$

$\frac{4}{4}$

$2^{\phi(125)} \equiv 1 \mod 125$

$2^{\phi(5^3)} \equiv 1 \mod 125$

$2^{100} \equiv 1 \mod 125$

QUESTION 2. (4 points) (direct proof). Let x, y be odd integers. Prove that $x + y + 49$ is an odd integer.

Suppose that x, y are odd. Then there exists $m, n \in \mathbb{Z}$ such that $x = 2m+1$ and $y = 2n+1$. Then we have that $x+y+49 = 2m+2n+51$ which can be written as $2(m+n+25)+1$ which is by definition odd as $m+n+25 \in \mathbb{Z}$. ∎

$\frac{4}{4}$

QUESTION 3. (4 points) (contradiction) Let x be an irrational number. Prove that $7x + 10$ is irrational.

Suppose that x is irrational. Now assume that $7x+10$ is rational. This implies that $7x+10 = \frac{p}{q}$ for some $p, q \in \mathbb{Z}$, $q \neq 0$. This then implies that $x = \frac{p}{7q} - \frac{10}{7} = \frac{p-10q}{7q}$, which is a contradiction as x is irrational. $7x+10$ must be irrational. ∎

$\frac{4}{4}$

QUESTION 4. (8 points) Find the smallest positive integer x such that $x\ (mod 21) = 11$ and $x\ (mod\ 5) = 4$. Show the work.

Note that $\gcd(21,5) = 1$, hence we may apply the Chinese remainder theorem.

$x \equiv \underset{r_1}{11} \pmod{\underset{m_1}{21}}$

$x \equiv \underset{r_2}{4} \pmod{\underset{m_2}{5}}$

1. Let $n_1 = \frac{21 \cdot 5}{21}$ $\boxed{= 5.}$

$5 \mod 21 = 5.$

$\Rightarrow 5^{-1}$ in $\mathbb{Z}_{21}$ $\boxed{= 17}$

$\frac{85}{5} = 17$

2. Let $n_2 = \frac{5 \cdot 21}{5}$ $\boxed{= 21}$

$21 \mod 5 = 1$

1^{-1} in $\mathbb{Z}_5$ $\boxed{= 1.}$

$\therefore$

$x = 5 \cdot 17 \cdot 11 + 21 \cdot 1 \cdot 4 \pmod{21 \cdot 5}$

$= \cancel{169}\ 1019 \pmod{105}$

$\boxed{x = \cancel{64}\ 74}$

$\frac{8}{8}$

Name AHMAD SAAD, ID 00897123

Quiz III, MW

Ayman Badawi

20/20

QUESTION 1. (10 points) Use the 4th method and prove that $\sqrt{15}$ is an irrational number.

Deny. $\sqrt{15}$ is rational.

$\sqrt{15} = \frac{a}{b}$ $\quad 15 = \frac{a^2}{b^2}$ $\quad a^2, b^2 \in \mathbb{Z}$ $b \neq 0$

$\cancel{a^2, b^2}$ a^2 is odd, a is odd

b^2 is odd, b is odd

$a = 2n+1$ $\quad b = 2m+1$

$m, n \in \mathbb{Z}$

$15 = \frac{(2n+1)^2}{(2m+1)^2}$

$15(4m^2+4m+1) = 4n^2+4n+1$

$4 \cdot 15m^2 + 4 \cdot 15m + 15 = 4n^2+4n+1$

$\frac{4 \cdot 15m^2 + 4 \cdot 15m + 14}{4} = \frac{4n^2+4n}{4}$

10/10

$\underbrace{15m^2+15m+7/2}_{\text{not integer}} = \underbrace{n^2+n}_{\text{integer}}$ A contradiction. Hence, $\sqrt{15}$ is irrational.

QUESTION 2. (10 points) Use math induction and prove $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$. i.e, show that $\frac{1}{2} + \frac{1}{6} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ $n \geq 1$

1- Prove for smallest n, $n=1$. $\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{2}$, $\frac{n}{n+1} = \frac{1}{1+1} = 1/2$ $\quad 1/2 = 1/2$.

2- Assume $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ for some $n \geq 1$

3. $\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \sum_{i=1}^{n} \frac{1}{i(i+1)} + \frac{1}{(n+1)(n+2)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)+1}{(n+2)(n+1)}$

↳ Replace from step 2.

$= \frac{n^2+2n+1}{(n+2)(n+1)}$

$= \frac{(n+1)^2}{(n+2)(n+1)} = \frac{n+1}{n+2}$

Proved $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ by induction.

10/10

Name AHMAD SAAD, ID 89723

20/20.

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Quiz IV, MW

Ayman Badawi

QUESTION 1. (**7 points**) Use math induction and prove $6 \mid [n(n+2)(n+4)]$, for every EVEN integer $n \geq 2$.

1- Show true for smallest n, n=2

$2(4)(6) = 6(8)$, factor of 6.

2- Assume $6 \mid [n(n+2)(n+4)]$ for some even integer $n \geq 2$

3- Prove.

$(n+2)(n+4)(n+6) = \underbrace{n(n+2)(n+4)}_{\text{factor of 6 by step2.}} + \underbrace{6(n+2)(n+4)}_{\text{factor of 6}}$

Hence, proved by math induction.

7/7.

QUESTION 2. (**6 points**) Use truth table and show $\overline{s_1 \Rightarrow s_2} \equiv s_1 . \overline{s_2}$.

S_1	S_2	$S_1 \Rightarrow S_2$	$\overline{S_1 \Rightarrow S_2}$	$S_1 . \overline{S_2}$
1	1	1	0	0
1	0	0	1	1
0	1	1	0	0
0	0	1	0	0

6/6

QUESTION 3. (**7 points**) Use math induction and prove $\sum_{i=1}^{n}(4i+3) = n(2n+5)$, for every $n \geq 1$.

1- Prove for smallest n, n=1

$\sum_{i=1}^{1} 4i+3 = 7$, $1(2+5) = 7$ $7=7$

2- Assume $\sum_{i=1}^{n}(4i+3) = n(2n+5)$ for some $n \geq 1$.

3- Proof $\sum_{i=1}^{n+1}(4i+3) = (n+1)(2n+7)$

$\sum_{i=1}^{n+1}(4i+3) = \underbrace{\sum_{i=1}^{n}(4i+3)}_{\text{Replace by \#2}} + \overset{4n+7}{\cancel{4n+3}} = n(2n+5) + 4n+7$

$= 2n^2 + 9n + 7$

$= (n+1)(2n+7)$. Hence, proved by math induction

7/7.

Name Nawaf Alabdi, ID b00093759

Quiz VI, MW

Ayman Badawi

20/20

QUESTION 1. (8 points)
Let $A = \{2, \{2\}, 3, 5\}$, $B = \{3, 5, 6, \{3\}\}$. Find

(i) $A \cup B =$.

$\{2, \{2\}, 3, 5, 6, \{3\}\}$

(ii) $A \cap B =$

$\{3, 5\}$

8/8

(iii) $A - B =$

$\{2, \{2\}\}$

(iv) $B - A =$ $\{6, \{3\}\}$

QUESTION 2. Let $A = \{3, \{1, 3\}, \{3\}\}$

a) (**3 points**) Write down all elements of $P(A)$.

$\phi = \{\}$, $\{3\}$, $\{\{1,3\}\}$, $\{\{3\}\}$, $\{3, \{1,3\}\}$, $\{3, \{3\}\}$,

$\{\{1,3\}, \{3\}\}$, $\{3, \{1,3\}, \{3\}\}$ 3/3

b) (**9 points**) Write T or F

(i) $\{3\} \in A$ T

9/9.

(ii) $\{3\} \subseteq A$ T

(iii) $\{1, 3\} \in P(A)$ F

(iv) $\{3, \{1, 3\}\} \subseteq P(A)$ $3 \notin P(A)$. F.

(v) $\{\{3\}, \{1, 3\}\} \in P(A)$ T

(vi) $\{\phi, \{3\}\} \subseteq P(A)$. $\{3\} \in P(A)$ & $\phi \in A$. T

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Name Fatima Alzarooni, ID 94188

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$\frac{19}{20} + 1 = \frac{20}{20}$

Quiz V, MW

Ayman Badawi

QUESTION 1. (9 points) Write down T OR F

each = 1.5

(i) $\exists! x \in Z$ such that $\forall\, y \in Z$, $yx^2 - 9y = 0$. F

$x = 3$ $x = -3$ $9y - 9y = 0$

(ii) $\exists\, x \in Z$ such that $2x - 5 = 0$ iff $3y + 5 = 0$ for some $y \in Q$. F

$S_1 = F$ $x = \frac{5}{2} \notin Z$ $S_2 = T$ $y = \frac{-5}{3} \in Q$

(iii) $\forall\, y \in Z, \exists\, x \in Z$ such that $3x + y = 0$ iff $w^2 - 2 = 0$ for some $w \in Z$. T

$S_1 = F$ $x = \frac{-y}{3} \notin Z$ $S_2 = F$ $w = \pm\sqrt{2} \notin Z$

(iv) $\forall y \in Q, \exists\, x \in Z^*$ such that $xy \in Z$. (note $Z^* = Z - \{0\}$) T

(v) $\exists\, x \in Z$ such that $\forall\, y \in Z$, $2xy - 6y = 0$ T

$x = 3$ $6y - 6y$

(vi) If $2x^2 - 8 = 0$ for some $x \in Z$, then $w^2 - 5 = 0$ for some $w \in R$. T

$S_1 = T$ $S_2 = T$

$\frac{9}{9}$

QUESTION 2. (10 points)

$$For\ k = 6\ to\ n^3 + 7$$
$$x = 7 * k^5 + 3 * k^3 - 10 \qquad 8 \to * \quad 2 \to +/-$$
$$For\ i = 1\ to\ k$$
$$y = x^7 + 2 * i^4 \qquad 10 \to * \quad 1 \to +$$
$$Next\ i$$
$$Next\ k$$

a) Find the exact number of the arithmetic operations that the code will execute.

b) Find O(code)

outer loop	inner loop
# of times it runs: $(n^3+7-6)+1 = (n^3+2)$ times	# of times it runs: K times
# of arithmetic op.: $10(n^3+2)$	# of arithmetic op.: $11K$

$66, \ldots, 11(n^3+7)$

# of total arithmetic op: $(n^3+2)\left[\frac{66 + 11(n^3+7)}{2}\right] + 10(n^3+2)$

$O(\text{code}) = n^6$

$\frac{10}{10}$

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Quiz VI, MW

Ayman Badawi

20/20

QUESTION 1. (8 points)
Let $A = \{2, \{2\}, 3, 5\}$, $B = \{3, 5, 6, \{3\}\}$. Find

(i) $A \cup B =$.

$\{2, \{2\}, 3, 5, 6, \{3\}\}$

(ii) $A \cap B =$

$\{3, 5\}$

8/8.

(iii) $A - B =$

$\{2, \{2\}\}$

(iv) $B - A =$ $\{6, \{3\}\}$

QUESTION 2. Let $A = \{3, \{1, 3\}, \{3\}\}$

a) (**3 points**) Write down all elements of $P(A)$.

$\phi = \{\}$, $\{3\}$, $\{\{1,3\}\}$, $\{\{3\}\}$, $\{3, \{1,3\}\}$, $\{3, \{3\}\}$,
$\{\{1,3\}, \{3\}\}$, $\{3, \{1,3\}, \{3\}\}$

3/3

b) (**9 points**) Write T or F

(i) $\{3\} \in A$ T

9/9.

(ii) $\{3\} \subseteq A$ T

(iii) $\{1, 3\} \in P(A)$ F

(iv) $\{3, \{1, 3\}\} \subseteq P(A)$ $3 \notin P(A)$. F.

(v) $\{\{3\}, \{1, 3\}\} \in P(A)$ T

(vi) $\{\phi, \{3\}\} \subseteq P(A)$. $\{3\} \in P(A)$ & $\phi \in A$. T

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Quiz VII, MW

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20/20

QUESTION 1. (14 points)
Given (*) $a_n = 6a_{n-1} - 9a_{n-2}$, where $a_1 = 0$, $a_2 = -9$.
(a) Find a general formula for a_n

12 points

HLR:

$a_n - 6a_{n-1} + 9a_{n-2} = 0$

$\dfrac{\alpha^n - 6\alpha^{n-1} + 9\alpha^{n-2}}{\alpha^{n-2}} = 0$

$\alpha^2 - 6\alpha + 9 = 0$

$\alpha_1 = 3 \quad \alpha_2 = 3$

$a_h = c_1(3)^n + c_2 n(3)^n$

$a_1 = 3c_1 + 3c_2 = 0$

$a_2 = 9c_1 + 18c_2 = -9$

$a_n = (1)3^n - (1)n(3)^n$

$\boxed{a_n = 3^n - n(3^n)}$ → general formula

by calculator:

$c_1 = 1 \qquad c_2 = -1$

14/14

2 points (b) Use (*) and find a_3. Then use (a) and find a_3.

$a_3 = 3^3 - (3)(3^3) = -54$

(*) $a_3 = 6a_2 - 9a_1 = 6(-9) = -54$

QUESTION 2. (6 points) Let $a_n = 6a_{n-1} - 9a_{n-2} + 12n + 18$. Find a general formula for a_n. Note that $a_h(n)$ is the same as in $Q1$. So find $a_p(n)$. Do not find c_1, c_2.

particular:

$a_p = 12n + 18 = an + b$

$a_n - 6a_{n-1} + 9a_{n-2} = 12n + 18$

$an + b - 6(a(n-1) + b) + 9(a(n-2) + b) = 12n + 18$

$an + b - 6(an - a + b) + 9(an - 2a + b) = 12n + 18$

$an + b - 6an + 6a - 6b + 9an - 18a + 9b = 12n + 18$

$4an + 4b - 12a = 12n + 18$

$4a = 12 \qquad 4b - 12a = 18 \rightarrow 4b = 54$

$a = 3 \qquad b = \frac{27}{2}$

$a_p = 3n + \frac{27}{2}$

$a_n = a_h + a_p$

$\boxed{a_n = 3^n - n(3)^n + 3n + \frac{27}{2}}$

6/6

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20/20.

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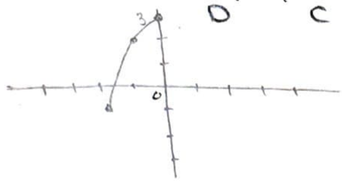
Quiz VIII, TR

Ayman Badawi

QUESTION 1. (10 points)
Let $f : [-2, 0] \to [-1, 3]$ such that $f(x) = -x^2 + 3$. DRAW the CURVE.

D C

x	-2	-1	0
y	-1	2	3



range $= [-1,3]$

a) Is f ONTO? explain briefly

onto $\Rightarrow$ range = codomain

$[-1,3] = [-1,3]$

Yes it is onto because range of f is equal to the codomain

b) Is f one-to-one? explain briefly.

By Horizontal line test $\forall b \in C$ $y=b$ intersects the curve at exactly one point so it is one-to-one.

c) If yes for (a) and (b), find the inverse of f and find the domain and the range of f^{-1}.

$f(x) = -x^2+3$

$\boxed{f^{-1}(x) = -\sqrt{3-x} \quad f^{-1} : \underset{D}{[-1,3]} \to \underset{C}{[-2,0]}}$

$f^{-1}(x) \Rightarrow y = -x^2+3 \Rightarrow x = -y^2+3 \quad y^2 = 3-x$

$y = \pm\sqrt{3-x}$ we need to choose 1 we choose $-\sqrt{3-x}$ because it lies the range of f^{-1} $[-2,0]$

QUESTION 2. (10 points) Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 7 & 8 & 1 & 5 & 6 & 2 \end{pmatrix}$

a) Find the smallest integer n such that $f^n = I(x)$

$(1\ 3\ 7\ 6\ 5) \circ (2\ 4\ 8)$
5-cycle 3-cycle

smallest $n = \text{LCM}[5,3] = \frac{5 \times 3}{\gcd(5,3)} = \frac{15}{1}$

$n = 15$

$5 \div 3 = 1$ remainder 2; $3 \div 2 = 1$ remainder 1; $2 \div 1 = 2$ remainder 0

b) Find the smallest integer m such that $f^m = f^{-1}$, and find f^{-1}

$f^{-1} = f^m = f^{n-1} = f^{14}$ $m = 14$

$f^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 8 & 1 & 2 & 6 & 7 & 3 & 4 \end{pmatrix}$